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Magnetohydrodynamic flows and turbulence: a report on the Second Bat-Sheva Seminar

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This paper is a summary of the Second Bat-Sheva Seminar on magnetohydrodynamic (MHD) Flows and turbulence. It was held in the University of the Negev, Israel, on 28–31 March 1978, with 64 participants from 7 countries. Reviews and research papers were presented on the general theory of MHD, MHD duct flows (with emphasis on novel aspects such as non-uniform fields and fluid properties, bends, free-surface effects and longitudinal diffusion), two-phase flows (especially those likely to occur in a liquid-metal generator), turbulence and instabilities, and electrically driven flows (with new results presented for the theory of laminar and turbulent flows in induction furnaces, and for the theory of thermo-electrically driven flows in transverse magnetic fields). One day of the conference was devoted to turbulence, mainly without magnetic fields, with reviews and new results presented on the theory and measurements of coherent structures, intermittency at high Reynolds number, methods of calculating shear flows, and measurement techniques. The seminar was a strange mixture of people and topics, which produced some interesting papers and some useful discussion.

1. Introduction

The Second Bat-Sheva Seminar was held at Ben-Gurion University, Beer-Sheva, Israel, to bring together a disparate group of people working in liquid-metal magnetohydrodynamics, turbulence and some associated areas of fluid mechanics. It was the aim of the conference organizer (H. B.) to select participants whose research related to the work done by his group at Beer-Sheva. The full proceedings of this conference will eventually be published (in about two years), but it seemed worthwhile to publicize sooner the new findings and the discussion at this conference.

Following the recent conference on MHD held at Grenoble in 1976 (Hunt & Moreau 1976) and at Beer-Sheva in 1975 (Branover 1976), there was considerable emphasis at this conference on how new ideas and techniques in liquid-metal MHD could be applied, and it was in that sense a continuation of the central theme of the Euromech conference at Grenoble.

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The conference opened with a stimulating review of 'Some analogous problems in fluid mechanics and MHD' by *Shercliff* (Warwick). The mathematical analogy between different phenomena and different fluid and electromagnetic properties was particularly stressed. Shercliff explained how he had found these analogies to be very helpful in both researching and expounding on MHD. The classical analogy between fluid mechanics and MHD that is now always used for illustrative purposes (but was missed in the original research) involves the conservation theorems of Kelvin for the flux of vorticity in an inviscid fluid and of Alfvén for the flux of magnetic flux density B in a perfectly conducting fluid. Shercliff practised his preaching by describing some important analogies between two-dimensional inviscid rotational flow satisfying $\nabla^2\psi = f(\psi)$ (cf. Shercliff 1977) and a two-dimensional static plasma satisfying $\nabla^2 A = f(A)$, where the analogous functions $\psi(x,y)$ and $A(x,y)$ are the stream functions (or vector potentials) of the flow and the magnetic field respectively. A plasma problem whose solution exploits this analogy is that of finding the external electric currents which will confine a given plasma shape containing given currents. The positions of the former often turn out to be along branch cuts in the complex plane!

In the ensuing discussion a number of useful analogies were pointed out: Ludford mentioned how recent analysis of an elastic membrane in torsion has exploited MHD duct-flow analysis; Moffatt pointed out that in some ways $\boldsymbol{\omega}$ and $\mathbf{B}$ are analogous, but there are important differences between them because of the connexion between vorticity and velocity (see, for example, Moffatt 1978*b*). Duct flows in strong transverse magnetic fields and in a rotating system have some interesting similarities which were exploited by Walker (1975*a*, *b*).

For future reference, essential non-dimensional numbers for MHD flows are the Reynolds number $Re = u_0 L\rho/\eta$, the Hartmann number $M = B_0 L(\sigma/\eta)^{\frac{1}{2}}$, the interaction parameter $N = M^2/Re$ and the magnetic Reynolds number $R_m = \mu\sigma u_0 L$, where $u_0, L, B_0, \rho, \eta, \sigma$ and μ are the velocity scale, length scale, magnetic flux density, density, viscosity, conductivity and magnetic permeability respectively. A parameter of importance in MHD duct flow is the ratio $\Phi = \sigma_\omega t/\sigma a$ of the conductance of the walls to that of the fluid, σ_ω being the wall conductivity and t the thickness.

2. MHD Duct flow; laminar and free-surface flows

Ludford† (Cornell) & Walker (Illinois) gave a review lecture on 'Current status of MHD duct flow', in which they gave a detailed account of the present state of the theory of laminar incompressible flows in ducts with strong transverse magnetic fields at low magnetic Reynolds number. The theory for uniform magnetic fields and variable-area ducts is fairly well developed, and has been previously reviewed (Walker & Ludford 1977).

Ludford maintained that since the last Bat-Sheva seminar progress on flows with streamwise magnetic field variation has been slow. In reviewing these problems he showed that quite a number of different duct flows could be analysed in such fields, provided that the magnetic field was planar, e.g. $\mathbf{B} = (B_x(x,y),\ B_y(x,y),\ 0)$. He expected little or no progress with three-dimensional fields because an orthogonal co-ordinate system parallel and perpendicular to the field is then not in general possible. The new solution presented by Ludford was that of a constant-area, circular,

† The author whose name is in italics gave the lecture.

highly conducting duct in a non-uniform transverse field, to which only an approximate solution by Holroyd (1976) had been available hitherto (see also Holroyd & Walker 1978; Hunt & Holroyd 1977).

Holroyd & *Hunt* (Cambridge) reviewed 'Theoretical and experimental studies of liquid-metal flow in strong non-uniform magnetic fields in ducts with complex geometry'. The main object of their review was to show how many of the qualitative, and in some cases quantitative, features of MHD duct flow in non-uniform fields and along pipe bends, manifolds, area changes, etc. can be predicted by plotting out the 'characteristic surfaces', along which the fluid tends to travel. (These are defined by constant values of the integral $\int |\mathbf{B}(s)|^{-1} ds$ along field lines in the fluid between the duct walls; see Hunt & Moreau 1976; Holroyd & Walker 1978; Kulikovskii 1973.) The conditions for these surfaces to define the flow are that the conductance of the walls Φ be very small (e.g. non-conducting or very thin and slightly conducting) and that the magnetic field be strong enough for inertial forces to be negligible. (In the sense that a general qualitative understanding was now available, Hunt disagreed with Ludford's opinion that progress had been slow!)

As well as measurements of the velocity and pressure along straight pipes in non-uniform magnetic fields, where the theory is reasonably well confirmed, new experimental results were presented for flow in a double pipe bend (⎽⎽│‾‾) with the intermediate leg parallel to the magnetic field, the wall conductance being small. There is a pressure drop at each bend and a negligible pressure gradient in the pipe parallel to the magnetic field. From the method of characteristic surfaces, all flow at the bends should move to the walls (parallel to the field). Bocheninskiĭ, Tananaev & Yakovkev (1977) have also measured velocity profiles in such a flow around a bend, and obtained M-shaped velocity profiles (i.e. velocity profile maxima occurring in boundary layers on walls parallel to the field) for an interaction parameter around 8 (not inertialess), as predicted above.

Holroyd & *Hunt* also described their calculation and measurements of the pressure drop along straight ducts with thin conducting walls which have a length $2a$ in the direction of $\mathbf{B}$ much greater than their length $2b$ in the other direction. It was found that if $a/b = 4{\cdot}75$ a long rectangular and diamond-shaped cut increases the ratio of flow rate per unit area per unit pressure gradient by a factor of 3 compared with a circular duct. It may be worth thinking about inserting a matrix of such ducts inside a circular duct in a liquid-metal-cooled fusion reactor!

Walker (Illinois) & *Ludford* (Cornell) presented some new calculations on liquid-metal flows in open channels under the action of a uniform magnetic field transverse to the flow, the channels having constant width and conducting side walls (see Walker 1975*b*). The analysis exposes some novel free-surface phenomena as the slope of the channel and the magnetic field are changed. The most interesting effects occur when the governing parameter G is of order 1, where $G = \rho g/\sigma B_0^2 u_0$, the ratio of gravitational to electromagnetic forces; also $G = 1/(F^2 N)$, F being the Froude number and N the interaction parameter. The authors showed that the free-surface profiles were analogous to those found by hydrologists analysing unconfined aquifers (Pavlovskiĭ 1956). These analyses may possibly be of use in estimating the behaviour of liquid metal in foundries when it is pumped up a slope or slowed down when flowing down a slope.

McNab (Westinghouse, Pittsburgh) gave an account of various experiments to measure the pressure rise along a d.c. pump using a strong transverse magnetic field, the pump being a rectangular duct filled with sodium potassium eutectic. Typical duct sizes were 18 × 56 × 200 m, with velocities in the range 1–10 m s^{-1}, $M = 600$ and N in the range 0·5–50. The conducting walls were copper coated with nickel. The object of the experiments was to measure and, if possible, improve the efficiency of liquid-metal d.c. pumps, which, because of their reliability, are being considered for pumping in fast breeder reactors (cf. Davidson & Thatcher in Hunt & Moreau's 1976 review). In these experiments a duct efficiency of 65 % was achieved (in larger ducts now being considered at Westinghouse and Argonne it is possible that this figure may be increased to 80 %). At these high Hartmann numbers the improvement of duct efficiency is limited by the short-circuiting of electric currents and distortion of the flow field at each end of the electrodes and each end of the magnet's pole pieces. Fluid dynamically, the most interesting feature of McNab's results was the sharp *decrease* in pressure of the flow on entering the pump and the sharp *increase* on leaving, the opposite of what is found and calculated at low values of the interaction parameter (Shercliff 1962, pp. 70–76). An approximate theory has been developed for these unexpected distortions.

Branover, Hoch, Landsberg, Unger & Zilberman (Beer-Sheva) also discussed this problem in their paper on the effect of fringe fields on flow in rectangular channels with different aspect ratios. The special feature of their work was the measurement of the turbulence in the entrance region. The duct was non-conducting and its dimensions were 20 mm parallel to **B** and 48 mm perpendicular. M and N were large. They found that near the entrance to the duct imposing a magnetic field *increased* the turbulence, but further downstream, when M-shaped profiles developed, the turbulence was reduced. As N was increased further the maxima in these side-wall boundary layers increased (as $N \rightarrow \infty$, all the flow occurs in the side walls when **B** is non-uniform (Ludford & Walker q.v.)) and velocity fluctuation persisted in this layer. It is planned to extend the work to study entrance effects in a toroidal channel.

Tabeling (Laboratoire de Génie Mécanique, Paris) & Chabrerie (Laboratoire de Génie Eléctrique, Université de Paris VI, XI) described a general method which used a double Fourier series and numerical integration to calculate the core flows and boundary layer profiles in MHD duct flow with arbitrary conductivities of the walls parallel and perpendicular to the uniform magnetic field. The analysis is valid when $M \geqslant 1$ and the cross-stream width is much larger than the thickness of any side-wall layer. Comparisons were made with all published results for particular wall conductivities and agreement was found to be excellent. The calculations can be extended to narrow ducts, and it is found that if the ratio (cross-stream width/side-wall-layer thickness) $\gtrsim 2{\cdot}5$ the asymptotic theory applies (e.g. Temperley & Todd 1971).

P. Hervé (Réunion) presented theoretical results for Couette flow in a toroidal duct with axis vertical in a transverse (axial) magnetic field, the flow being driven by the motion of a conducting disk on the liquid surface. Bendaoud (1974) considered the case of insulating side walls, while Hervé treated arbitrary wall conductivities. Hervé supposed that the Hartmann number was large and found that, except when the cross-stream thickness is very small compared with the height of the torus, the ratio Φ of the wall to the fluid conductance has little effect on the speed of the core flow. This confirms some experimental results of Vivès (1975).

Yakhot, Hoch, Levin & Branover (Ben-Gurion University and Technion, Haifa) gave a theoretical description of the effect on non-conducting streamwise baffles on the current obtained from an MHD generator channel without considering dynamical effects. Because the current return paths near the edge of the channel are disturbed by the baffles the net current output can increase for a given velocity field U down the channel. The analysis can be done for quite general baffle configurations by conformal-mapping techniques. It was pointed out by Shercliff that it was not obvious that improved efficiency could be obtained in this way since the extra viscous loss induced by the baffle might more than compensate for the increase in current (Shercliff 1975). Pierson confirmed that at Argonne they had thought about this expedient, but had rejected it; the cure was worse than the problem itself.

Schouten (Delft) gave an approximate analysis of entrance flow into an MHD duct.

Moffatt's (Bristol) analysis of 'Streamwise diffusion in MHD duct flow' began with an extension of Taylor's (1953) laminar flow analysis of longitudinal diffusion to the case of the streamwise diffusion of temperature in a laminar flow of liquid metal along a rectangular channel in which the boundaries are conducting and there is a transverse magnetic field B_0. Moffatt found that when $B_0 = O\,(M = 0)$ the effect of non-zero wall conductivity is to increase the velocity of the centroid of a convected temperature distribution and to *decrease* its longitudinal dispersion relative to the centroid. When a magnetic field is applied which flattens the velocity profile, the longitudinal diffusion is, not surprisingly, reduced (cf. Erdogan 1969). The theory can also be used to calculate the distortion and perhaps amplification of a weak magnetic field by liquid-metal pipe flows, which for example occur in fast breeder reactors. The addition of swirl may be important. Pierson (Argonne) commented that, following Bevir's (1973) speculation on how a magnetic field might be amplified by a large quantity of liquid metal in motion in a system of pipes, his group had done their own calculations and decided that self-generation of a magnetic field was not at all likely (Pierson 1975).

3. MHD turbulence and instability

3.1. *Turbulent flow and laminarization*

Lykoudis (Purdue University, Indiana) described improvements to a semi-empirical theory for turbulent MHD flows which he had developed previously (Lykoudis 1967; Lykoudis & Brouillette 1967). The Reynolds-stress term in the equation, which was modelled previously by a term involving the unknown skin friction c_f, is now given by the expression $\exp\{-AM^2/Re^{1\cdot75}\}$, where A is a constant and Re the Reynolds number. Use of this form for the stresses yielded good experimental agreement for mean velocity profiles, skin friction and Reynolds stresses for both transverse and aligned magnetic fields. Lykoudis also suggested that a useful empirical formula for M_t, the Hartmann number corresponding to the transition from turbulent to laminar flow, was $M_t = A_1(Re - Re_0)^{0\cdot875}$, where A_1 is a constant and Re_0 is the transition Reynolds number in the absence of a magnetic field.

The problem of relaminarization was taken up by Branover, *Gershon* & Zilberman (Beersheba), who described further results in their continuing study (Branover & Gershon 1976; Branover 1977) of the origin of residual disturbances in laminarized

MHD duct flows. They used a hot-film anemometer in mercury to find r.m.s. values of longitudinal velocity pulsations, their two-point correlations, and energy spectra at different locations in the channel ($6 \times 30\ \mathrm{mm}^2$ cross-section) and for different positions of the electromagnet supplying the field. The main conclusion was that the residual disturbances that still appear after relaminarization originate as two-dimensional disturbances of the M-shaped profile near the entrance to the channel rather than as turbulence carried in from the field-free region. This is also suggested by the relationship between the disturbance level and the interaction parameter based on wall thickness parallel to the magnetic field.

The twin themes of secondary disturbances and anisotropy induced by the field were taken up in two further papers. Chabrérie & *Tabeling* (Paris) investigated flows of liquid metals driven electrically by radial currents in an annulus in the presence of an axial magnetic field. M was large, and an important parameter was the ratio c of the width of the section to the radius of the torus. When Mc is large, a perturbation analysis shows that there are both Hartmann eddies of amplitude $O(M^{-5})$ and secondary eddies of amplitude $O(M^{-4})$. This secondary flow can distort the primary flow, but the effect is negligible if $c^{\frac{1}{2}}\, ReM^{-\frac{5}{4}} \ll 1$, where Re is the Reynolds number. Experiments carried out for $CM^{\frac{1}{2}} \gg 1$ (side-wall layers much thinner than cross-stream width) confirm this criterion. If $CM^{\frac{1}{2}} \ll 1$, on the other hand, there is a transition to Taylor-vortex flow in agreement with the work of Chandrasekhar (1961) and others. Recently the authors extended their experiments to include the use of NaK, which has a kinematic viscosity 15 times higher than that of mercury. This enabled the secondary flow to be studied with more accuracy.

Sulem and Frisch (Observatoire de Nice) and Alemany and *Moreau* (Institut de Mécanique de Grenoble) described experimental and theoretical investigations of homogeneous MHD turbulence at low magnetic Reynolds numbers (Alemany *et al.* 1978). An external magnetic field produces an anisotropic, wavenumber-independent dissipation; its strength is characterized by the interaction parameter N_0, defined as the ration of the large-eddy turnover time t_0 to the Joule dissipation time. An experiment reported had interaction parameters of the order of unity. Turbulence was created by a grid moving down through a column of mercury (diameter 0·2 m, height 2·7 m) in the presence of a vertical magnetic field (0–0·25 T) produced by a solenoid. Reynolds numbers of the turbulence equal to several hundred were obtained. The velocity $v_{\parallel}$ parallel to the magnetic field was measured with quartz-coated hot-film probes attached to the grid. A $k_{\parallel}^{-3}$ energy spectrum and a $t^{-1 \cdot 7}$ decay law for the energy were observed. A quasi-steady equilibrium between energy transfer and Joule dissipation was proposed as an explanation. This differs from the theory of Moreau (1968), which predicts a $k_{\parallel}^{-2}$ spectrum (see Hunt & Moreau 1976).

The case $N_0 \gg 1$ was analysed theoretically. For isotropic or moderately anisotropic initial conditions, the Joule dissipation was found to inhibit strongly the build-up of triple correlations. Hence a dissipation-dominated linear phase was found (Moffatt 1967; Schumann 1976). This linear phase lasted up to a time t_{NL}, where $t_{NL} \simeq t_0 N_0^{\frac{1}{3}}$. At later times $t > t_{NL}$, the energy was confined to nearly transverse wave vectors in a cone $|\theta - \frac{1}{2}\pi| \lesssim \epsilon \simeq N_0^{\frac{1}{3}}$, where θ is the angle with the external field. In a thought experiment where the external field was suddenly switched off, isotropy was restored in a time $t_{3D} \simeq t_{NL} \log(1/\epsilon)$, by a mechanism analogous to the migration of errors from small to large scales in the predictability problem for strictly two-dimensional flows (Leith & Kraichnan 1972). Actually, the nonlinear phase is not governed by the laws of two-dimensional turbulence; Joule dissipation and energy transfer are comparable, and their competition again produces a quasi-steady equilibrium. Further insight is obtained by the application of a standard two-point closure. In particular, a universal range was found in which the angular spectrum follows a ϕ^{-9} law, where $\phi = \frac{1}{2}\pi - \theta$.

3.2. *Instability*

M. Garnier & J. Garnier (Institut de Mécanique de Grenoble) investigated the stability of an interface between two fluids (one of them insulating) in the presence of an alternating magnetic field (with frequency $\omega \sim 10^3$–10^4 Hz). The frequency was supposed to be so high $[(B_0^2/\rho U_0)(\sigma/\mu\omega)^{\frac{1}{2}} \ll 1]$ that the $\mathbf{j} \wedge \mathbf{B}$ forces are quasi-steady and confined to a thin layer at the interface. In contrast to results by the same authors presented in Hunt & Moreau (1976) it was claimed that the action of the electromagnetic forces is typically *destabilizing*. Although disturbances with wave vectors normal to the field can be stabilized by field gradients, those with vectors parallel to the field are typically destabilized by the same gradients.

Plaschko (Technische Universitat, Berlin) also discussed a free-surface problem: the instability of the free surface of a liquid-metal jet with a parallel magnetic field. It was supposed that the jet width varied slowly downstream and the basic profile was chosen from a single-parameter family. Spatial rather than temporal evolution of the disturbance was considered. It was found that the effect of the field was to inhibit instability just behind the orifice and to favour non-aligned disturbances, thus providing, as noted by Hunt, yet more evidence of the irrelevance of Squire's theorem to this type of problem.

Fautrelle (Institute de Mécanique de Grenoble) discussed baroclinic instability in the presence of a magnetic field (Fautrelle 1978). If the buoyancy frequency $\tilde{N}$ is much less than the rotation frequency $|\mathbf{\Omega}|$, baroclinic modes have high wavenumbers at right angles to $\mathbf{\Omega}$ according to the Taylor–Proudman theorem and this leads to a simplified set of equations allowing diffusion to be included. Although the effect of the applied field is typically stabilizing, the range of unstable wavelengths is increased for weak fields, especially when the field is asymmetric with respect to the midplane of the layer. Calculations were done in the linear regime for weak fields and a nonlinear analysis was performed for strong fields. It was found that a sufficiently strong field suppresses the instability entirely. The magnetic field, while providing no energy itself, acts to release part of the potential energy of the system into the baroclinic waves. The study was motivated by the possibility that the upper part of the earth's core is stably stratified (Kennedy & Higgins 1973) but Fautrelle recognized that the evidence for this is far from conclusive.

4. Two-phase flow in liquid-metal MHD generators

Trovillion, *Kurzweg*, Elkins & Lindgren (Gainesville, Florida) described their analysis of fully developed laminar flow in a rectangular channel with conducting side walls parallel to B_0 and insulating top and bottom walls. The viscosity and conductivity were prescribed non-uniform functions of position across the duct (there was no dynamical or thermodynamic determination of these functions). The

Hartmann number was large. These features were chosen to help predict the two-phase flow in the liquid-metal MHD generators being studied at Argonne National Laboratory.

From numerical solutions, with variable grid spacing near the wall, it was found that if the conductivity was assumed highest near the walls (corresponding to the greater concentration of liquid there) the boundary layers were wider and the flow tended to travel in a jet down the centre. Calculations are now under way to discover whether baffles can ensure a more uniform distribution of the flow.

Lykoudis (Purdue) gave an analysis of liquid-metal MHD generators with a shunt layer. Experiments at Argonne show that two-phase generators there tend to have thin layers of pure liquid on the walls with vapour (whose conductivity σ_2 may be smaller by a factor of 10^{-4} than that of the liquid σ_1) occupying the centre. The efficiency of these generators can be seriously reduced if the slow moving liquid layers act as a short circuit. Lykoudis described a fully developed laminar flow analysis for an MHD duct flow, to work out the effects of these two layers. He concluded that their behaviour is determined by a modified Hartmann number $M_l = M[(\eta_1/\eta_2)(\sigma_1/\sigma_2)]^{\frac{1}{2}}$, where M is based on the vapour phase and η_1/η_2 is the ratio of the viscosities. In practice $M \sim 5000$ and $M_l \lesssim \frac{1}{2}M_1$; Lykoudis showed that in this case if the thickness of either layer is less than one-tenth of the duct width they are not a serious practical problem. A similar two-layer analysis with heat addition has been performed by Owen, Hunt & Collier (1976).

Dunn, *Fabris*, Pierson & Petrick (Argonne National Laboratory, ANL) presented a brief summary of the two-phase liquid-metal MHD power cycles, their method of operation, and their performance potential. Next, a brief summary of the ANL generator experiments was given, with a statement that these data are available to other researchers for comparison with analytical models (write to Prof. Pierson). The high temperature (to 810 °K) sodium–nitrogen generator test facility and the substitute circular-pipe test section were described. For the flow of pure liquid in a circular pipe the measurements of the normalized resistance coefficients λ^*_{sp}, proportional to the pressure gradient with a magnetic field minus the pressure gradient for no magnetic field, agreed well with the value derived from the theory of Chang & Lundgren (1961), namely

$$\lambda^*_{sp} = \frac{2M_{sp}}{Re_{sp}}\left(\frac{M_{sp}\,\Phi'_{sp}+1}{\Phi'_{sp}+1}\right),$$

where Φ'_{sp} is the wall-to-fluid conductance ratio with the contact resistance included. For high interaction parameters the data fell below the laminar-theory line, and this is believed to be due to M-shaped velocity profiles developed in the entrance region. The theory was extended to two-phase flows, and the least-squares fit of the data agreed with the extended theory to within 9 % except for high-void-fraction cases. Experimental data from the ambient-temperature rectangular generator also agree well with the theory, viz. to within 20 %.

Fabris, Dunn & Pierson (Argonne National Laboratory) first discussed two-phase flow and foams in liquid-metal MHD generators, and the ANL programme to generate foams using surface-active agents. The gas–liquid mixer was discussed next as it influences the generator. The present mixer has been shown in air–water tests to give less than optimum performance (large bubbles, inhomogeneous flow) either with pure (tap) water or with water plus soap (a surface-active agent). New designs have been tested with porous elements that yield small bubbles with pure water and even smaller bubbles with soap. A conceptual design for a mixer has been developed, tests are under way, and fabrication is planned for this year.

Hot-film and resistivity probes have been tested first with air–water (mixer measurements) and then with NaK and N. With hot-film probes air and water bubbles can be distinguished clearly, and in pure NaK the damping of the turbulence with increasing magnetic field strength is seen. For NaK and N with no magnetic field single bubbles can clearly be seen and as the field strength is increased the number of bubbles appears to decrease everywhere but is highest near electrodes. For the resistivity probe a similar decrease is observed. For open-circuit conditions (positive pressure gradient) the number of bubbles appears to increase. The data are still preliminary at this stage.

Elkins, *Lindgren*, Trovillion & Kurzweg (Gainesville, Florida) discussed two-phase MHD channel flow. Lindgren is interested in bubbly flow, and neglected the gas equations because it was not known how to incorporate them. The governing equations for the liquid were averaged in terms of measurable quantities, and could be solved if the void fraction was known from measurements. In a channel flow, if there is little or no turbulence the viscous force is important only in the wall layer and the core flow is determined by the current and magnetic flux densities. From the equations, the velocity is higher where the void is higher and this leads to large slip ratios (ratio of average gas to average liquid velocity). The dynamical similarity of these air–water tests to liquid-metal two-phase flow was discussed.

5. Electromagnetically driven motions

5.1. *Stirring problems*

The important problem of electromagnetic stirring, relevant to induction furnace technology, was considered in three related papers. In a review lecture on 'MHD problems in alternating magnetic fields', *Moreau* (Institut de Mécanique de Grenoble) gave an account of the use of rapidly alternating fields to stir and melt metal. Three basic approaches were underlined. In a global approach (Tir 1965) the force field is prescribed and the dissipation, free-surface shape, etc., may be evaluated. The pattern of the flow depends on the ratio L/δ, where L is the size of the container and δ the skin depth. One can attack the problem numerically by simply solving the induction equation for the field B (which is not affected by the motion as R_m is small) and thus finding $\mathbf{J} \wedge \mathbf{B}$. Unfortunately there is not good quantitative agreement with experiments. Finally a phenomenological approach can be adopted, where the system is idealized (e.g. Sneyd 1971) and a complete solution, one hopes, yields general qualitative criteria. The structure of the free-surface perturbation in a furnace can also be investigated: the perturbation turns out to be negligible for low frequencies.

It is difficult to make an accurate model of the force field structure since it is not clear under what circumstances the fluctuating part of the $\mathbf{J} \wedge \mathbf{B}$ force is unimportant. *Hunt* & Maxey (Cambridge) considered this problem. $\mathbf{J} \wedge \mathbf{B}$ has a steady and a fluctuating part, which both have magnitude of order $B_0^2/\mu\delta$ and the latter of which has frequency 2ω, where ω may be anything from 3000 to 1 Hz. If $\mathbf{u}$ is divided into mean ($\bar{\mathbf{u}}$), fluctuating ($\tilde{\mathbf{u}}$) and turbulent ($\mathbf{u}$) parts, then three regimes can be distinguished: (i) if $|\bar{\mathbf{u}}| \gg |\tilde{\mathbf{u}}|$ then the flow is effectively steady; (ii) if $|\bar{\mathbf{u}}| \approx |\tilde{\mathbf{u}}|$ and the eddy or integral time scale τ_I is such that $\tau_I \gg \omega^{-1}$ then rapid-distortion analysis is necessary (e.g. Townsend 1976); (iii) if $\tau_I \ll \omega^{-1}$ then the turbulence 'turns over' in a time much less than that in which **B** changes as the flow responds quasi-steadily to the forcing. However, a closer analysis reveals that

$$|\tilde{\mathbf{u}}|/|\bar{\mathbf{u}}| \sim \frac{B_0}{(\mu\rho)^{\frac{1}{2}}} \frac{\kappa}{\omega L} \left(\frac{L}{\delta}\right)^{\frac{1}{2}},$$

where κ is the Kármán constant, and this is likely to be very small in most applications. It was concluded that the quasi-steady or empirical theories apply in general, as has been assumed by several investigators (e.g. Moreau q.v.).

Flow can also be generated by rotating rather than alternating fields, as described in a review talk by *Moffatt* (Bristol). These motions can be used to measure electrical conductivity for hot metals and other cases where handling is difficult (Ozelton & Wilson 1966), as a stirrer (Kapusta 1968), and as a generator of droplets in a swirling spray (Alemany & Moreau 1977). The laminar regime can be successfully analysed (Moffatt 1965), especially for circular cylinders in the low Hartmann number regime. Circular Couette flow is produced except for very rapid rotation rates, when there is a thin magnetic boundary layer and the bulk of the fluid rotates as a solid body. Moffatt made the important point that for laminar flow with closed streamlines the force balance *must* be between Lorentz and viscous forces, as shown by integrating round the streamlines (e.g. Batchelor 1956) (though in a turbulent flow there is a balance between $\mathbf{J} \wedge \mathbf{B}$ forces and Reynolds stresses, which are independent of viscosity. It was also shown that, for the circular-cylinder problem, flow with circular streamlines is possible only if the Hartmann number is not too large, and that then there are *two* possible steady-state solutions for the core angular velocity.

5.2. *Technological applications*

Block (RWTH Aachen) described (with a film) the use in a steel works of an inductor to transport steel uphill for 2 m from a container to a $140 \times 140\ \text{mm}^2$ mould. The advantages of this process are (i) improved quality due to stirring of the metal while it is pumped, (ii) safety, (iii) easier access since there is no ladle above the mould and (iv) good control: a method has been developed to give constant speed of withdrawal.

A rather different application was discussed by *Thatcher* (Risley Laboratory) in a talk on sodium electrotechnology at the Risley Nuclear Power Development Laboratory. He described current work on flow couplers: a liquid-metal MHD pump generator in one liquid-metal loop coupled electrically but not mechanically to a similar device in another loop. The purpose of the device is to reduce thermal shocks in an intermediate heat exchanger by coupling the flow in the two loops. If one loop fails, some flow is maintained in the second loop by electromagnetic forces. Outstanding problems include the effects of field distortion, and the increase in pressure drop in the pipe due to armature reaction. Experiments show that $\Delta p \simeq \frac{1}{2} R_m \Delta p_0$, where Δp_0 is the pressure drop in the absence of a field. Work is in progress in stages to design a full-sized coupler with emphasis on the understanding of scaling laws and of the geometry of the pipes carrying the secondary flow. *Shercliff*, Alty & Dutta Gupta (University of Warwick) described how thermoelectric effects, caused by very large temperature differences, may be usable to help pump lithium in the cooling blanket of a fusion reactor. Both the Thompson and the Peltier effects could be important. (A good reference for the continuum approach to the subject is Woods 1975.) If a temperature difference is applied across a stainless-steel container containing lithium, then for speeds of the order of 25 m s^{-1}, heating rates due to the Peltier effect can be of the order of 0·5 MWm^{-2}, so the boundary conditions have to be modified. Some experiments are being done at Warwick to investigate the problem. The thermoelectric effect due to impurities in liquid metals may also be usable to increase mixing in metal processing. A noteworthy point is that strong magnetic fields B may produce *less* rather than more motion, as the thermoelectric force varies like B whereas the drag varies like B^2.

6. Turbulence, waves and convection

6.1. *Turbulent flows*

In a review talk on 'Marginal stability of turbulent flow' *Lessen* (Office of Naval Research) pointed out that many quasi-steady turbulent flow problems (such as turbulent wakes) have gross features that can be predicted by considering the linear marginal stability problem with a modified (turbulent) viscosity η_T. The consequent relation $\eta_T/\eta \simeq R/R_{\mathrm{cr}}$, where R is the Reynolds number and R_{cr} the critical value of R for marginal instability (Landau & Lifshitz 1959, p. 120), was, Lessen suggested, not generally correct for flows with subcritical instabilities but a good guide for systems that are supercritically unstable, such as a convection layer. He illustrated this idea with experimental observations of annular Couette flow and of the breakdown of a vortex sheet, and commented on his own work on the statified shear layer (Lessen, Barcilon & Butler 1977). Some of the audience questioned the use of *marginal* stability theory. Lessen replied that in these supercritical systems the turbulence increases the rate of dissipation of energy until it just balances the energy created by the instabilities. Any more dissipation and there could be no quasi-steady configuration!

In contrast to this global approach, *Klebanoff* (National Bureau of Standards, Washington) & Frenkiel (Naval Ship Research and Development Center, Bethesda) reported experiments on the small-scale structure of turbulence under various conditions in wind and water tunnels downstream from a grid, and in the boundary layer on a flat plate. Probability density distributions and high-order moments of the temporal gradients of the turbulent velocity were reported. The Kolmogorov theory (based on a lognormal probability distribution) predicts the relationship

$$\frac{\langle(\partial U/\partial t)^{2n}\rangle}{\langle(\partial U/\partial t)^2\rangle^n} \propto R_\lambda^{\frac{3}{4}\mu n(n-1)},$$

where R_λ is the turbulent Reynolds number and μ is a universal constant given by the exponent of the dissipation correlation function. One main conclusion of the study is that this dependence on n of the slope of the probability density distribution is not compatible with μ being independent of R_λ. It was noted in discussion that Kraichnan (1974) and Mandelbrot (1974) have seriously questioned the lognormality assumption.

The small-scale approach was also emphasized by *Amini* (Institut de Mécanique de Grenoble), who studied the development of a 'laminar spot' generated by a local perturbation in a laminar boundary layer. A turbulent spot in a flat-plate boundary layer was created artifically by a short pulse injected into the boundary layer through a small hole in the plate. To investigate the condition for the growth of a spot, attention was paid to the laminar stage of the disturbance i.e. at points upstream from the turbulent spot (but downstream from the injection point) where pulse-triggered ensemble averages were taken of the streamwise velocity U. The free stream velocity was $U_0 \sim 6$ m/s and the hole was 0·3 m from the leading edge and of diameter 1 mm. Visualization was achieved by dye injection. It was found that the short pulse triggers a vortex ring and a horseshoe vortex, which lifts up, breaks up and gives rise to another ring further downstream. The triggered ring moves more slowly than the vortex and finally disappears. A detailed picture of the development of the spot was obtained from hot-wire measurements.

The influence of initial conditions on the development of the turbulent mixing layer was studied by Oster and *Wygnanski* (Tel Aviv University) and Dziomba and Fiedler (Technische Universität Berlin). Small amplitude oscillations were introduced at the trailing edge of a splitter plate separating two streams of unequal velocities. This increased the spreading rate of the mixing layer dramatically and self-similarity was attained only far downstream (when $x \sim \text{constant} \times U_c/f$, where U_c is the velocity of large eddies and f the frequency). The axisymmetric jet (Crow & Champagne 1971) is rather similar; mixing appears to be due to the roll-up of the vortex sheet into large eddies. The coherent large-scale structure appears to be two-dimensional as seen in the pictures of Brown & Roshko (1974). Strong external disturbances were introduced to check the persistence of these eddies. For $Re \simeq 10^6$ they seem stable and so are unlikely to be a relic of the transition process as was suggested by Chandrsuda *et al.* (1978).

More theoretical aspects of the problem were addressed by *Wolfshtein* (Technion, Haifa), who discussed modelling of the diffusion of turbulence energy. The triple velocity correlation, which constitutes the main part of the diffusion, may be represented by gradients of the Reynolds stresses in all three space directions. For boundary-layer flows the diffusion may be expressed in terms of turbulence-energy gradients, but the turbulent Prandtl number for the turbulence energy is dependent on the structure of the flow and may vary with the spatial position. It was also argued that in more complex flows the diffusion cannot be expressed in terms of turbulence-energy gradients. The following references are also relevant: Naot, Sharit & Wolfshtein (1974); Wolfshtein, Naot & Lin (1975); Lin & Wolfshtein (1977).

Naot (Center for Technological Education, Holon, Israel) dealt with thermodynamic aspects of anisotropic turbulence using numerical models. The main goal of the discussion was to demonstrate the possibility of using a second-law analysis, in its statistical form, for the closure procedure used to derive the coefficients of the stress-transport turbulence model (Costa de Beauregard & Tribus 1974; Naot 1977, 1978; Wolfshtein *et al.* 1975).

Fabris (Argonne National Laboratory) described the use of a special four-wire probe to investigate the turbulent wake of a cylinder and its interaction with an equal wake, and *Ledermann* (Polytechnic Institute of New York) gave a spirited exposition of the application of laser scattering techniques to fluid flow diagnostics. Classically educated fluid dynamicists in the audience were surprised to discover that anti-Stokes lines appear in this discipline too!

6.2. *Convection and waves*

Geffen & Lustman (Department of Applied Mathematics, Tel Aviv University) investigated the propagation and breakdown of magnetoacoustic waves in a conducting compressible gas. Since the identification of the starting point and the estimation of the characteristic time for shock formation, for solution of nonlinear equations with smooth initial conditions, are in general still unsolved problems, initial data with weak discontinuities (i.e. discontinuities in the first derivatives) were considered. For a hyperbolic quasi-linear system of equation, such singularities must be located on a characteristic surface (the wave front), and the jump in the gradient is completely determined by a scalar quantity (Lustman & Geffen 1977). Under some conditions the equation satisfied by this scalar quantity reduces to an ordinary differential equation, the solution of which can become infinite at certain critical points and times (Jeffrey & Taniuti 1969). An infinite jump in the gradient (a break in the wave) may be thought of as the beginning of a shock wave, where the solution itself becomes discontinuous. This analysis was applied to the cases of slow and magnetoacoustic waves.

Proctor (Cambridge) gave an account of the initial convective instability of liquid metals. Jones, Moore & Weiss (1976) had previously shown that low Prandtl number ($\sigma = \nu/\kappa$, where ν is the kinematic viscosity and κ the thermal diffusivity) fluids could convect significant amounts of heat only if the Rayleigh number (a dimensionless measure of the conductive heat flux) was about 140 % of its classical value for the onset of instability. Below this critical value, reducing the Prandtl number to zero suppresses convection entirely, but above it an asymptotic limit is possible for which the heat flux is independent of the Prandtl number. Proctor gave an analytical solution to the problem in a horizontal cylinder (Proctor 1977), and showed how the form of the flow is determined, first by the property that vorticity is constant on streamlines and second by the balance on each streamline between viscous and thermal forces; inertial forces do not contribute in this case, as was pointed out earlier in a different context by Moffatt in his review lecture (q.v.).

7. MHD power systems

Three interesting papers were given on the technical economic viability of some novel kinds of MHD power generation. These papers were not fluid dynamical and will not be summarized here, but it might be mentioned that all the authors seemed to think that such systems were unlikely to be used as power sources in the forseeable future. The papers were by Radebold (Technische Universität Berlin), 'A solar come-back of liquid-metal MHD systems', Tate & Zauderer (General Electric, Philadelphia), 'Self-excited pulsed MHD power generation', and Borda, Branover, Elbocher & Leitner (Beer-Sheva, Israel), 'On the possible use of MHD generators in solar energy systems'.

8. Conclusion

The conference concluded with a discussion of the basic problems in MHD that need investigating and the new problems raised by the applications of MHD in industry and energy generation.

Some of the basic problems specifically mentioned were the following.

(i) Inviscid flows in non-uniform magnetic fields, examples mentioned by Moffatt being flow through a magnetic field saddle point (i.e. $\mathbf{B} = (\alpha x, -\alpha y, 0)$) and past a line current, i.e. the dynamical extension of Cowley's (1961) field sweeping calculations.

(ii) Turbulence: why is it that the energy spectrum $E(k)$ of homogeneous turbulence in a magnetic field has the form $E \sim k^{-3}$, k being the wavenumber, which is the *same* form as that of two-dimensional turbulence, even though the turbulence is not two-dimensional? What are the effects of sidewalls on turbulence experiments? (Moreau). It would be useful if measurements of electric field fluctuations could be made to relate to velocity fluctuations (Klebanoff); these have been made independently, but not correlated. Pressure drops in turbulent MHD duct flow can be predicted by mixing-length arguments, but compared with the physical insight into non-magnetic turbulent shear flows or even density-stratified flows, MHD turbulence is poorly understood.

(iii) Two-phase flows: Moffatt suggested that in this field at least one should attempt to understand the effects of a magnetic field on a single bubble, to echo the call for understanding the forces on solid particles in MHD flows by Moreau at Grenoble (Hunt & Moreau 1976). However, Lykoudis said that might be quite pretty theoretically but would be of little use in actually predicting two-phase MHD duct flow, which he said needed to be understood. Given the paucity of theory and good measurements of two-phase flow with no magnetic field, it will be a long time before his plea is answered.

The discussion of the application of MHD raised some interesting points.

(i) Fusion power. Shercliff and Lykoudis emphasized that nowadays the main MHD problems connected with fusion power have nothing to do with liquid-metal cooling of the blanket. As a cooling method, it is now quite out of fashion. In Shercliff's view the main problems are concerned with magnetohydrostatics and dynamical problems of the plasma itself. An example where MHD theory is especially necessary is in understanding the 'reversed field pinch', where, as Moffatt pointed out, currents are generated parallel to **B**. Understanding this pinch may be helped by reference to the α-effect and considering the 'helicity' of turbulent motion (Moffatt 1978*a*).

(ii) Metallurgical industry. Moreau noted that the stirring and heating of liquid metals in induction furnaces are broadly understood, but he considered that the real problem is that of understanding the motions of solid non-conducting inclusions in the melt. The stirring motions and particularly the surface instabilities in electrolysis cells, e.g. those for refining aluminium, need more analysis. There are many problems associated with the newer uses of electromagnetic fields in this industry, such as the control of a free surface by a magnetic field in continuum casting (Block 1973), and pulversation and flow regulation (Alemany & Moreau 1977).

(iii) Other applications mentioned were (*a*) induction pumps, where the prediction of the loss in pressure rise due to end effects, surface friction, etc., is still not well understood, and (*b*) welding, where there continues to be uncertainty about the theoretical problems (see, for example, Moffatt 1978*b*).

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